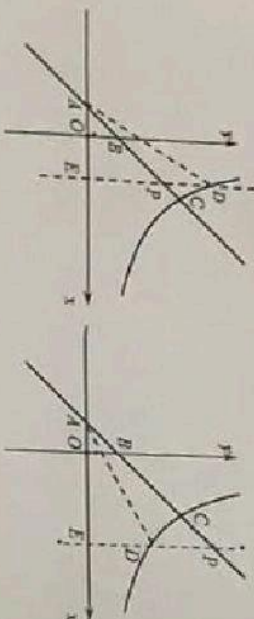


② 当点P在点D的上方时, $PD = y_P - y_D = a + 1 - \frac{6}{a}$. $\therefore \frac{a+1}{a+1-\frac{6}{a}} = 2$.

解得 $a = 3$ 或 -4 (舍去).
 $\therefore P_1(3, 4)$.

..... 9分
 10分

综上所述, 满足条件的点P的坐标为 $P_1\left(-\frac{1+\sqrt{17}}{2}, \frac{1+\sqrt{17}}{2}\right)$ 或 $P_2(3, 4)$.



20. (本小题满分10分)

(1) 证明: 如图, 连接OB, $\because GB = GE$, $\therefore \angle GBE = \angle GEB$.

$\because \angle GEB = \angle DEC$,

$\therefore \angle GBE = \angle DEC$.

$\because OB = OC$, $\therefore \angle OBC = \angle OCB$.

$\because GD \perp AC$, $\therefore \angle CED + \angle ECD = 90^\circ$.

$\because \angle OBC + \angle GBE = 90^\circ$,

$\therefore \angle OBG = 90^\circ$, $\therefore OB \perp BG$, $\therefore OB$ 为半径.

$\therefore HG$ 是 $\odot O$ 的切线.

【注: 不说明OB为半径, 扣1分】

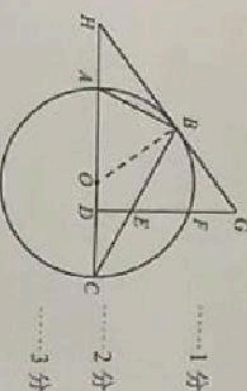


图1

(2) 解: i) 如图, 过点G作 $GP \perp BC$ 于点P,
 $\because BG = GE$, $GE = \frac{\sqrt{5}}{2} BE$, $\therefore PE = \frac{1}{2} BE = \frac{\sqrt{5}}{5} GE$.

$\therefore \cos \angle GEP = \frac{PE}{GE} = \frac{\sqrt{5}}{5}$, $\therefore \cos \angle DEC = \frac{\sqrt{5}}{5}$.

$\because \angle DEC + \angle ECD = 90^\circ$, $\angle BAC + \angle ECD = 90^\circ$,

$\therefore \angle DEC = \angle BAC$, $\therefore \cos \angle BAC = \frac{\sqrt{5}}{5}$.

$\therefore \frac{AB}{AC} = \frac{\sqrt{5}}{5}$.

图1

..... 4分

设 $AB = \sqrt{5}x$, $AC = 5x$, $\therefore BC = \sqrt{AC^2 - AB^2} = 2\sqrt{5}x$.

$\because \angle HBA + \angle GBE = 90^\circ$, $\angle GBE + \angle C = \angle DEC + \angle C = 90^\circ$, $\therefore \angle HBA = \angle C$, $\therefore \angle H = \angle H$.

$\therefore \triangle HBA \sim \triangle HBC$.

..... 5分

$$\frac{HA}{HB} = \frac{HB}{HC} = \frac{AB}{BC}, \therefore HB = \frac{4\sqrt{5}}{3}, \therefore \frac{HA}{4\sqrt{5}} = \frac{3}{4\sqrt{5}} \cdot \frac{1}{2}.$$

$$\therefore HC = \frac{8\sqrt{5}}{3}, HA = \frac{2\sqrt{5}}{3}, \therefore AC = HC - HA = 2\sqrt{5}.$$

$\therefore \odot O$ 的半径为 $\sqrt{5}$.

..... 6分

ii) 解法一: 如图, 连接CF.

$\because AF \parallel HG$, $\therefore \angle HBA = \angle BAF$.

$\because \angle HBA = \angle ACB$, $\therefore \angle BAF = \angle ACB$. 又 $\because \angle BAF = \angle BCF$, $\therefore \angle ACB = \angle BCF$.

$\therefore AC = 2\sqrt{5}$, $\cos \angle BAC = \frac{\sqrt{5}}{5}$, $\therefore AB = 2$, $BC = 4$, $\tan \angle BAF = \tan \angle ACB = \frac{1}{2}$.

$\therefore BM = AB \cdot \tan \angle BAF = 1$.

$\therefore CM = BC - BM = 3$.

$\therefore MF = CM \cdot \sin \angle BCF = CM \cdot \sin \angle ACB = \frac{3\sqrt{5}}{5}$.

$\because MF \parallel BG$, $\therefore \triangle MEF \sim \triangle BEG$.

$\therefore \frac{ME}{BE} = \frac{MF}{BG} = \frac{MF}{GE}$, $\therefore \frac{ME}{MF} = \frac{BE}{GE} = \frac{2\sqrt{5}}{5}$.

$\therefore ME = \frac{2\sqrt{5}}{5} MF = \frac{2\sqrt{5}}{5} \times \frac{3\sqrt{5}}{5} = \frac{6}{5}$.

$\therefore CE = CM - ME = 3 - \frac{6}{5} = \frac{9}{5}$, $\therefore CD = CE \cdot \sin \angle CED = \frac{2\sqrt{5}}{5} CE = \frac{18\sqrt{5}}{25}$.

$\therefore OD = OC - CD = \sqrt{5} - \frac{18\sqrt{5}}{25} = \frac{7\sqrt{5}}{25}$.

..... 10分

ii) 解法二: 连接BO交AF于点N, $\because AF \parallel HG$, $\therefore \angle HBO = \angle ANO = 90^\circ$, $\therefore OB \perp AF$.

$\because OB$ 为半径, $\therefore AF = 2AN$, $\because AB = 2$, $\tan \angle BAF = \frac{1}{2}$,

$\therefore AN = \frac{4\sqrt{5}}{5}$, $BN = \frac{2\sqrt{5}}{5}$.

$\therefore AF = 2AN = \frac{8\sqrt{5}}{5}$.

$\because OB = \sqrt{5}$, $\therefore ON = OB - BN = \sqrt{5} - \frac{2\sqrt{5}}{5} = \frac{3\sqrt{5}}{5}$.

$\because \angle ANO = \angle FND = 90^\circ$, $\angle NAO = \angle FND$, $\therefore \triangle NAO \sim \triangle FND$.

$\therefore \frac{AO}{AN} = \frac{FN}{AN}$, 即 $\frac{\sqrt{5}}{8\sqrt{5}} = \frac{FN}{AD}$, $\therefore AD = \frac{32\sqrt{5}}{25}$.

$\therefore OD = AD - AO = \frac{32\sqrt{5}}{25} - \sqrt{5} = \frac{7\sqrt{5}}{25}$.

..... 10分

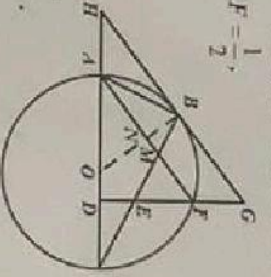


图2

..... 9分

B卷 (共50分)

一、填空题 (本大题共5个小题, 每小题4分, 共20分)

21. 6 22. $\frac{3}{8}$ 23. $\frac{1}{2}$ 24. $-\frac{15\sqrt{3}}{4}$ 25. $8\sqrt{2} - 8$; 4.

二、解答题 (本大题共3个小题, 共30分)

26. (本小题满分8分)

解: (1) 设篮球的单价为x元, 足球的单价为y元, 根据题意, 得

$$\begin{cases} 50x + 10y = 4600, \\ 30x + 30y = 4200. \end{cases}$$

..... 2分

$$\text{解得} \begin{cases} x = 80, \\ y = 60. \end{cases}$$

..... 4分

答: 篮球的单价为80元, 足球的单价为60元.

成都市武侯区 2021 年九年级第二次诊断性检测试题
数学参考答案及评分意见

A 卷 (共 100 分)

一、选择题 (本大题共 10 个小题, 每小题 3 分, 共 30 分)

题号	1	2	3	4	5	6	7	8	9	10
答案	A	C	B	D	D	C	B	C	A	B

二、填空题 (本大题共 4 个小题, 每小题 4 分, 共 16 分)

11. -1 12. a^2b 13. $\frac{5}{2}$ 14. $2\sqrt{3}$

三、解答题 (本大题共 6 个小题, 共 54 分)

15. (本小题满分 12 分, 每题 6 分)

- (1) 解: 原式 $= -3 + 2\sqrt{3} - 4 + 5 - 2\sqrt{3}$
 $= -2$ 4 分

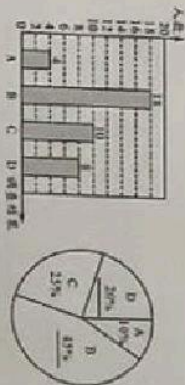
- (2) 解: 解不等式①, 得 $x > -2$.
解不等式②, 得 $x < 4$.
 $\therefore$ 不等式组的解集为 $-2 < x < 4$
 $\therefore$ 不等式组的非负整数解为: 0, 1, 2, 3. 6 分

16. (本小题满分 6 分)

- 解: 原式 $= \frac{x}{x+2} \cdot \frac{(x+2)^2}{x(x-3)}$
 $= \frac{x+2}{x-3}$ 4 分

17. (本小题满分 8 分)

解: (1) 如图所示:



- (2) $\frac{18}{4+18+10+8} \times 500 = 225$ (名). 【或 $45\% \times 500 = 225$ (名).】 5 分

答: “比较了解”的学生有 225 名.
(3) 列表如下:

	A_1	A_2	B_1	B_2
A_1	(A_1, A_1)	(A_2, A_1)	(B_1, A_1)	(B_2, A_1)
A_2	(A_1, A_2)	(A_2, A_2)	(B_1, A_2)	(B_2, A_2)
B_1	(A_1, B_1)	(A_2, B_1)	(B_1, B_1)	(B_2, B_1)
B_2	(A_1, B_2)	(A_2, B_2)	(B_1, B_2)	(B_2, B_2)

共有 12 种等可能性的结果, 其中抽到一男一女的结果有 8 种.

$$\therefore P = \frac{8}{12} = \frac{2}{3}$$

答: 恰好抽到一男一女的概率为 $\frac{2}{3}$ 8 分

18. (本小题满分 8 分)

解法一: 如图, 延长 DH 交 AB 于点 M , 设 $AM = x$ 米.

根据题意, 可知 $\angle AHF = 45^\circ$, $\angle EDF = 22^\circ$, $AE = 110$ 米, $DH = CG = 50$ 米, $BM = CD = 1.3$ 米.

在 $Rt\triangle AHM$ 中, $MH = \frac{AM}{\tan 45^\circ} = \frac{x}{1} = x$ (米). 2 分

$$\therefore EM = AM - AE = x - 110 \text{ (米)}.$$

在 $Rt\triangle DEM$ 中, $EM = DM \cdot \tan \angle EDF = (x + 50) \cdot \tan 22^\circ$
 $\approx 0.4(x + 50) = \frac{2}{5}x + 20$ 4 分

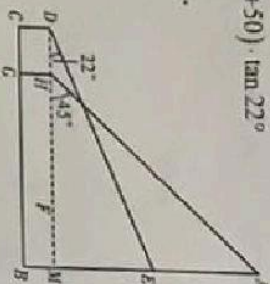
$$\therefore x - 110 = \frac{2}{5}x + 20$$

$$\therefore x = \frac{650}{3}, \text{ 即 } AM = \frac{650}{3} \text{ (米)}.$$

$$\therefore AB = AM + BM = \frac{650}{3} + 1.3 \approx 218 \text{ (米)}.$$

答: 双子塔 AB 的高度约为 218 米.

【注: 中间过程取一位小数, 即 $x \approx 216.7$, 不扣分.】



解法二: 如图, 延长 DH 交 AB 于点 M , 设 $EM = x$ 米.

根据题意, 可知 $\angle AHF = 45^\circ$, $\angle EDF = 22^\circ$, $AE = 110$ 米, $DH = CG = 50$ 米, $BM = CD = 1.3$ 米.

$$\therefore AM = AE + EM = x + 110$$

在 $Rt\triangle AHM$ 中, $HM = \frac{AM}{\tan \angle AHF} = \frac{x + 110}{\tan 45^\circ} = x + 110$ (米). 2 分

$$\therefore DM = DH + HM = x + 160 \text{ (米)}.$$

在 $Rt\triangle DEM$ 中, $EM = DM \cdot \tan \angle EDF = (x + 160) \cdot \tan 22^\circ$

$$\approx 0.4(x + 160) = \frac{2}{5}x + 64.$$

$$\therefore x = \frac{2}{5}x + 64$$

$$\therefore x = \frac{320}{3}, \text{ 即 } EM = \frac{320}{3} \text{ (米)}.$$

$$\therefore AB = AM + BM = \frac{320}{3} + 110 + 1.3 \approx 218 \text{ (米)}.$$

答: 双子塔 AB 的高度约为 218 米.

【注: 中间过程取一位小数, 即 $x \approx 106.7$, 不扣分.】

19. (本小题满分 10 分)

解: (1) $\because C(2, n)$ 在 $y = x + 1$ 的图象上, $\therefore n = 2 + 1 = 3$.

$$\therefore C(2, 3).$$

$$\therefore k = 2 \times 3 = 6.$$

$$\therefore y = \frac{6}{x}.$$

(2) 设 $P(a, a+1)$, $\therefore D(a, \frac{6}{a})$, $\therefore PE = y_p = a + 1$.

$$\therefore \frac{S_{\triangle OPE}}{S_{\triangle OPD}} = 2, \text{ 且两个三角形为等腰三角形, } \therefore \frac{PE}{PD} = 2.$$

① 当点 P 在点 D 的下方时, $PD = y_D - y_p = \frac{6}{a} - (a + 1) = \frac{6}{a} - a - 1$, $\therefore \frac{a + 1}{\frac{6}{a} - a - 1} = 2$.

解得 $a_1 = \frac{-1 + \sqrt{17}}{2}$, $a_2 = \frac{-1 - \sqrt{17}}{2}$ (舍去). 6 分

$$\therefore P_1 \left(\frac{-1 + \sqrt{17}}{2}, \frac{1 + \sqrt{17}}{2} \right).$$

由根与系数的关系, 得 $10x = -\frac{10}{a}$, $\therefore x = -\frac{1}{a}$, $\therefore -x+10 = \frac{1}{a}+10$.

$\therefore$ 点 D 的坐标为 $(-\frac{1}{a}, \frac{1}{a}+10)$.

.....4分

解法一: 设抛物线的对称轴交直线 AB 于点 F .

当 $x=5$ 时, $y=-5+10=5$, $\therefore F(5,5)$.

$\therefore CF = |x_F - x_C| = |-25a - 5| = |25a + 5|$.

$\therefore CF \parallel BE$, $\therefore \triangle BDE \sim \triangle FDC$.

$$\therefore \frac{BE}{CF} = \frac{|x_F - x_C|}{|x_D - x_C|} = \frac{|\frac{1}{a} + 10 - (-\frac{1}{a})|}{|-\frac{1}{a} - (-\frac{1}{a})|} = \frac{|\frac{1}{a} + 10 + \frac{1}{a}|}{|\frac{1}{a} + 10 + \frac{1}{a}|} = 1.$$

$$\text{即 } \frac{BE}{|25a+5|} = \frac{1}{|5a+1|}.$$

$$\therefore BE = (25a+5) \times \frac{1}{5a+1} = |5(5a+1)| \times \frac{1}{5a+1} = 5.$$

$\therefore$ 点 E 的坐标为 $(0,5)$.

【注: D 点坐标 2 分, 求 E 点坐标的过程中, 没有分类或没有用绝对值表示长度, 扣 1 分】

.....7分

解法二: 设直线 CD 的表达式为 $y = kx + m (k \neq 0)$.

由 $C(5, -25a)$, $D(-\frac{1}{a}, \frac{1}{a}+10)$, 得

$$\begin{cases} 5k + m = -25a \\ -\frac{1}{a}k + m = \frac{1}{a} + 10 \end{cases}$$

.....5分

利用加减消元, 整理得 $(5a+1)m = 25a+5 = 5(5a+1)$.

$\therefore D$ 不与 C 重合, $\therefore -\frac{1}{a} \neq 5$, $\therefore 5a+1 \neq 0$.

$\therefore m = 5$.

$\therefore$ 点 E 的坐标为 $(0,5)$.

.....7分

【注: 没有说明 $5a+1 \neq 0$, 不扣分】

(3) 解法一: 过 O 作 $OG \parallel CD$ 交 AB 于点 G , 过 D 作 $DH \parallel BO$, 交 x 轴于点 H , 连接 CG .

$\therefore ED \parallel OG$, $\therefore \frac{BE}{OE} = \frac{BD}{DG}$, $\therefore BE = OE$, $\therefore BD = DG$.

$\therefore OG \parallel CD$.

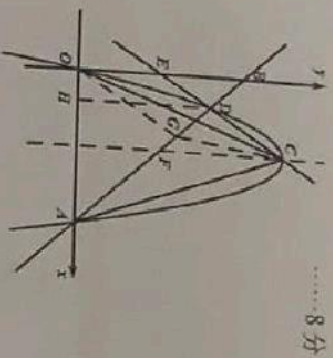
$\therefore S_{\triangle OEC} = S_{\triangle ODC}$ (同底等高).

① 如图, 当 $S_{\triangle OCB} = \frac{1}{4} S_{\triangle OCD}$ 时.

$$\therefore S_{\triangle OCB} = S_{\triangle ODC} = \frac{1}{4} S_{\triangle OCD}, \therefore \frac{S_{\triangle OBC}}{S_{\triangle OCD}} = \frac{DG}{AD} = \frac{1}{4}.$$

设 $DG = k$, 则 $DG = BD = k$, $AD = 4DG = 4k$.

$\therefore DH \parallel BO$, $\therefore \frac{OH}{OA} = \frac{BD}{BA} = \frac{k}{5k}$.



.....8分

$\therefore OH = \frac{1}{5} OA = \frac{1}{5} \times 10 = 2$, $\therefore$ 点 D 的坐标为 $(2, 8)$.

$\therefore D(2, 8)$ 在抛物线 $y = ax^2 - 10ax + 20a$ 上, $\therefore 8 = 4a - 20a$.

7

.....10分

$$\therefore a = -\frac{1}{2}.$$

② 如图, 当 $S_{\triangle OCB} = 4S_{\triangle OCA}$ 时.

$$\therefore S_{\triangle OCB} = S_{\triangle OCA} = 4S_{\triangle OCA}, \therefore \frac{S_{\triangle OCB}}{S_{\triangle OCA}} = \frac{DA}{DG} = \frac{1}{4}.$$

设 $DA = k$, 则 $BD = DG = 4DA = 4k$.

$\therefore DH \parallel BO$, $\therefore \frac{OH}{OA} = \frac{BD}{BA} = \frac{4k}{5k}$.

$\therefore OH = \frac{4}{5} OA = \frac{4}{5} \times 10 = 8$, $\therefore$ 点 D 的坐标为 $(8, 2)$.

$\therefore D(8, 2)$ 在抛物线 $y = ax^2 - 10ax + 20a$ 上,

$$\therefore 2 = 64a - 80a.$$

$$\therefore a = -\frac{1}{8}.$$

综上所述, a 的值为 $-\frac{1}{2}$ 或 $-\frac{1}{8}$.

.....12分

解法二: ① 如图, 当 $S_{\triangle OCB} = \frac{1}{4} S_{\triangle OCA}$ 时.

$$\therefore S_{\triangle OCB} = \frac{1}{2} OE \times (x_C - x_D) = \frac{5}{2} (5 + \frac{1}{a}), \quad S_{\triangle OCA} = \frac{1}{2} CF \times (x_A - x_D) = \frac{1}{2} (-25a - 5) \times (10 + \frac{1}{a}).$$

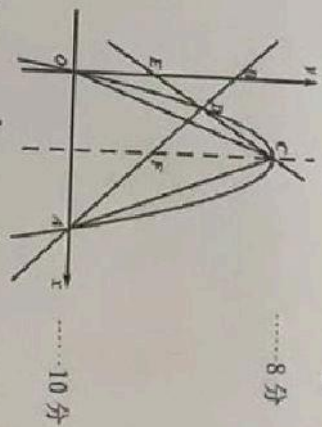
$$\therefore \frac{5}{2} (5 + \frac{1}{a}) = \frac{1}{4} \times \frac{1}{2} (-25a - 5) \times (10 + \frac{1}{a}). \quad (*)$$

整理, 得 $4(5a+1) = -(5a+1) \times (10a+1)$.

$\therefore D$ 不与 C 重合, $\therefore -\frac{1}{a} \neq 5$.

$$\therefore 5a+1 \neq 0, \therefore 4 = -10a-1.$$

$$\therefore a = -\frac{1}{2}.$$



.....8分

【注: (*) 部分, 也可以整理得 $10a^2 + 7a + 1 = 0$, 解得 $a_1 = -\frac{1}{2}$, $a_2 = -\frac{1}{5}$ (舍去)】

② 如图, 当 $S_{\triangle OCB} = 4S_{\triangle OCA}$ 时.

$$\therefore S_{\triangle OCB} = \frac{1}{2} OE \times (x_B - x_C) = \frac{5}{2} (-\frac{1}{a} - 5), \quad S_{\triangle OCA} = \frac{1}{2} CF \times (x_A - x_D) = \frac{1}{2} (5 + 25a) \times (10 + \frac{1}{a}).$$

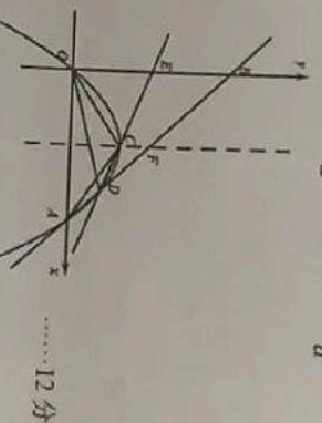
$$\therefore \frac{5}{2} (-\frac{1}{a} - 5) = 4 \times \frac{1}{2} (5 + 25a) \times (10 + \frac{1}{a}). \quad (*)$$

整理, 得 $-(5a+1) = 4(5a+1) \times (10a+1)$.

$\therefore D$ 不与 C 重合, $\therefore -\frac{1}{a} \neq 5$, $\therefore 5a+1 \neq 0$.

$$\therefore -1 = 4(10a+1).$$

$$\therefore a = -\frac{1}{8}.$$



.....12分

【注: (*) 部分, 也可以整理得 $40a^2 + 13a + 1 = 0$, 解得 $a_1 = -\frac{1}{8}$, $a_2 = -\frac{1}{5}$ (舍去)】

综上所述, a 的值为 $-\frac{1}{2}$ 或 $-\frac{1}{8}$.

【注: 一种情况下正确建立方程给 1 分, 一个 a 值 2 分, 两个 a 值 4 分】

8